An Extended N=1 Fermionic Supercurrent and its Associated N=2 Superconformal Algebra

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An extended free fermionic construction of the internal N=1 world sheet supercurrent for four-dimensional superstring theory is given. We show how it can describe theories with massless fermions, and we discuss the corresponding N=2 superconformal alg ebra. As an intermediate step, we show that an internal N=2 global superconformal invariance occurs in *any* superstring theory with massless fermions at tree level. To demonstrate this fact, we give the N=2 supercurrents for a model with N=1 space-ti me supersymmetry and a model without space-time supersymmetry.

1. Introduction

The internal conformal system of a superstring theory, and in particular the form of the world sheet superVirasoro generator, determine important properties of the low energy spectrum, including the presence of massless fermions and the Yang-Mills gauge g roup. In this letter, we give a new extended expression for the internal N=1 supercurrent for free fermions. It includes a background charge term as well as an additional term proportional to the world sheet fermions; and it is more general than both the standard supersymmetric Feigin-Fuchs construction[1] and the general form constructed out of 3n bosons[2]. Of course, two-dimensional field theories can be described in terms of either fermionic or bosonic fields whenever the zero modes of the bosonic fie lds have the values $\sqrt{2\alpha'}p \in Z+\nu$ where Z is the set of integers and ν is a rational number. In sect. 2, we derive the supercurrent, exhibit its hermitian form, and discuss its relevance to Yang-Mills gauge symmetries. In order to furth er analyze the role of this current in superstring models, we consider in sect. 3 the connection between world sheet properties and space-time properties. We show that *any* superstring theory with massless fermions at tree level has an N=2 internal superconformal algebra. This N=2 superconformal algebra corresponds to a global world sheet symmetry(not gauged) and so there is no corresponding ghost system. Although it is well known[3,4] that space-time supersymmetry requires an N=2 world sheet supers ymmetry, it is perhaps not so widely appreciated that string theories can have N=2 world sheet supersymmetry but not tree level space-time supersymmetry. To exemplify this feature, we give explicit constructions of the N=2 supercurrents for a model with s pace-time supersymmetry and for a model without space-time supersymmetry in sect's. 4 and 5. In sect. 6, we discuss how the new fermionic construction of the N=1

supercurrent can be used in theories with massless fermions and we compute its associated N= 2 supercurrents.

2. An Extended Realization of the N=1 superconformal algebra

The form of the N=1 supercurrent is critical in determining the particle spectrum of the string theory. Therefore, new constructions of the supercurrent may lead to more realistic string models. The operators in the internal superconformal field theory that we would like to consider are the world sheet fermions, $\psi^a(z)$ and the spin fields, $\Sigma^A(z)$. We must construct the supercurrent from various combinations of these fields and their derivatives so that the conformal weight is 3/2. A hint of what these operators might be is given by the operator product of spin fields $\Sigma^A(z)\Sigma^B(w) = (z-w)^{-3/4}\tilde{\psi}^{AB} + (z-w)^{1/4}(\frac{1}{3!}\tilde{\psi}\tilde{\psi}\tilde{\psi} + \frac{1}{2}\partial\tilde{\psi})^{AB} + ...$ where $\tilde{\psi} \equiv \sqrt{1/2}\Gamma_a^{AB}\psi^a(w)$ and products of $\tilde{\psi}$ are antisymmetric products of gamma matrices. These operator products contain similar combinations of operators to those in the supercurrent shown in (2.1). The supercurrent is:

$$T_F(z) = -\frac{i}{6}f_{abc}\psi_a(z)\psi_b(z)\psi_c(z) + \rho_a\partial_z\psi_a(z) + \sigma_a\psi_a(z)z^{-1} \tag{2.1a}$$

The corresponding Virasoro current is:

$$L(z) = \tfrac{1}{2}\partial_z\psi_a(z)\psi_a(z) + \rho_a\partial_z T_a(z) + 2\sigma_a T_a(z)z^{-1} + 2(\sigma^2 - \sigma_a\rho_a)z^{-2} \tag{2.1b}$$

where $T_a = -\frac{i}{2}f_{abc}\psi_b\psi_c$, and f_{abc} are the structure constants of a Lie group, $f_{abc}f_{abe} = 2\delta_{ce}$, and ρ and σ are arbitrary vectors subject to the restriction that $f_{abc}\rho_b\sigma_c = 0$; and we find that $c = \frac{\dim g}{2} - 12\rho^2$. These cons traints are required for closure of the N=1 algebra.

In physical contexts, one usually wants $L_n^\dagger = L_{-n}$ and $G_r^\dagger = G_{-r}$. This restriction is unnecessary mathematically but it can be met by the following constraints. With the supercurrent in the form $T_F(z) = \frac{1}{2}\sum_r G_r z^{-r-\frac{3}{2}}$, the hermiticity relation $G_r^\dagger = G_{-r}$ requires that ρ is pure imaginary and that the imaginary part of σ is one half ρ. These constraints can be written as $\mathrm{Re}\rho_a = 0$ and $\mathrm{Im}\rho_a = 2\mathrm{Im}\sigma_a$.

Combining the constraints for closure of the algebra with the constraints for hermiticity, we find the following condition must hold: $f_{abc}(\mathrm{Re}\sigma_b)(\mathrm{Im}\sigma_c) = 0$. The central charge is then given by $c = \frac{dimg}{2} + 12(\mathrm{Im}\rho)^2$.

The second term in (2.1a) provides a supersymmetric version of the Feigin-Fuchs free field construction of the Virasoro algebra [1]. The currents in (2.1) form an N=1 superconformal algebra and satisfy the following operator product expansions:

$$T_F(z)T_F(w) = \frac{c/6}{(z-w)^3} + \frac{\frac{1}{2}L(w)}{(z-w)} \quad ; \quad L(z)T_F(w) = \frac{\frac{3}{2}T_F(w)}{(z-w)^2} + \frac{\partial T_F(w)}{(z-w)}$$

$$L(z)L(w) = \frac{c/2}{(z-w)^4} + \frac{2L(w)}{(z-w)^2} + \frac{\partial L(w)}{(z-w)} \tag{2.2}$$

We now address the interesting question of how these currents will influence the gauge symmetry. The ρ and σ independent terms in (2.1a,b) have the standard Kac-Todorov mixing with the set of gauge generators $T_a(z)$[5]. This describes an alg ebra containing superKac-Moody generators with the appropriate conformal dimension. The terms proportional to ρ_a and σ_c in (2.1) break the symmetry to a subset of generators given by $\hat{\rho}_b T_b(z) + \frac{2}{z}\hat{\rho}_b \sigma_a \delta_{ba}$ where $f_{abc}\hat{\rho}_b \sigma_c = 0$, $f_{abc}\hat{\rho}_b \rho_c = 0$ and $\delta_{ab}\hat{\rho}_a \rho_b = 0$.

To describe the real world, any string theory must have massless fermions which then acquire a mass by symmetry breaking. In the next section, we investigate what constraints massless fermions put on the form this supercurrent.

3. N=2 superconformal algebra

An important question in string theory is whether or not space-time supersymmetry is required for a consistent finite theory. Efforts have been made to develop string models without space-time supersymmetry[3,4]. It is therefore useful to know what world sheet properties are required for a realistic model that may not have space-time supersymmetry. It has been shown that a global N=2 superconformal symmetry, which is global in the sense that it does not have a corresponding N=2 ghost system, is required f or space-time supersymmetry[6,7]. What about string models without space-time supersymmetry? We now point out from previous arguments that an N=2 global world sheet invariance follows directly from the presence of massless fermions at tree level. It then follows that theories which have N=2 superconformal symmetry do not necessarily have space-time supersymmetry. It turns out that there are at least three conditions that are required to guarantee space-time supersymmetry[7,8,9].These include N=2 superco nformal invariance, quantization of the U(1) charges, and a condition on the spectral flow operator.

The N=2 superconformal algebra is given by:

$$L(z)L(w) = \frac{\frac{1}{2}c}{(z-w)^4} + \frac{2L(w)}{(z-w)^2} + \frac{\partial L(w)}{z-w} + \ldots$$

$$L(z)T_F^{\pm}(w) = \frac{\frac{3}{2}T_F^{\pm}(w)}{(z-w)^2} + \frac{\partial T_F^{\pm}}{(z-w)} + \ldots \quad ; \quad L(z)J(w) = \frac{J(w)}{(z-w)^2} + \frac{\partial J(w)}{z-w} + \ldots$$

$$J(z)J(w) = \frac{\frac{1}{3}c}{(z-w)^2} + \ldots \quad ; \quad J(z)T_F^{\pm}(w) = \frac{\pm T_F^{\pm}(w)}{z-w} + \ldots$$

$$T_F^{+}(z)T_F^{-}(w) = \frac{\frac{1}{12}c}{(z-w)^3} + \frac{\frac{1}{4}J(w)}{(z-w)^2} + \frac{\frac{1}{4}L(w) + \frac{1}{8}\partial J(w)}{z-w} + \ldots$$

$$T_F^{\pm}(z)T_F^{\pm}(w) = 0 + \ldots \qquad (3.1)$$

From (3.1), it can be seen that the N=1 superconformal algebra is a subset of the N=2 superconformal algebra. This shows the relationship between the actual N=1 superconformal gauge algebra and the N=2 superconformal algebra.

We now consider a free N=1 superconformal field theory with fermions. The holomorphic part of the massless fermion vertex operator in the $-\frac{1}{2}$ picture is $V^{\alpha}_{-\frac{1}{2}}(z) =$

$S^\alpha(z)e^{-\frac{1}{2}\phi(z)}\Sigma(z)$. Here, S^α are spin fields of the Lorentz group SO(3,1) and Σ is a spin field of conformal weight 3/8 of the internal conformal field theory. The spin fields create the ground states of the Ramond or twisted sectors from the Neveu-Schwarz vacuum. In a free string theory, the spin field can be bozonized and put in the general form(omitting cocycles), $\Sigma(z) =: e^{i\nu_i\phi_i(z)}$: where, for fermionic descriptions, ν describes the boundary conditions of the world sheet fermions. For this argument, since we are only considering one massless fermion, the cocycles are irrelevant. The operator product of an internal spin field with its hermi tian conjugate contains a dimension one field J(z):

$$\Sigma(z)\Sigma^\dagger(w) = (z-w)^{-3/4} + (z-w)^{1/4}\tfrac{1}{2}J(w) + ... \tag{3.2}$$

which satisfies:

$$J(z)\Sigma(w) = \frac{\frac{3}{2}\Sigma(w)}{z-w} + ... \quad ; \quad J(z)\Sigma^\dagger(w) = \frac{-\frac{3}{2}\Sigma^\dagger(w)}{z-w} + ... \tag{3.3}$$

The derivation of (3.2) and (3.3) goes as follows. With the spin field in the bosonic form above, $\Sigma(z)\Sigma^\dagger(w) = (z-w)^{-\frac{3}{4}}[1 + (z-w)(i\nu_i\partial\phi_i(w)) + ...]$. The U(1) current, which is conventionally normalized as in (3.1),

is $J(z) = 2i\nu_i\partial\phi_i(z)$. Then, $J(z)\Sigma(w) = 2\nu_i\nu_i\frac{\Sigma(w)}{(z-w)} = \frac{\frac{3}{2}\Sigma(w)}{(z-w)}$ since $\frac{\nu_i\nu_i}{2} = h = 3/8$. If the internal conformal field theory is non trivial, the space-time supersymmetry algebra can be used to derive these operator products.

The U(1) current and spin field can be simplified by making a rotation of the bosonic fields ϕ_i to a basis $H, H_1, ...H_{i-1}$. We rotate the fields ϕ_i so that the spin field and U(1) current are functions only of the boson H. This requires tha t $H = \frac{2}{\sqrt{3}}\nu_i\phi_i$. So in the new basis, we have $J(z) = i\sqrt{3}\partial H(z)$, $\Sigma =: e^{i\frac{\sqrt{3}}{2}H}$:, and $\Sigma^\dagger =: e^{-i\frac{\sqrt{3}}{2}H}$:. (3.2) and (3.3) can also be derived in this new basis.

The N=1 supercurrent does not have a definite U(1) charge, but it can be split into two parts :

$$\begin{aligned} T_F(z) &= e^{i\sqrt{\frac{1}{3}}H}\tilde{T}_F^+(z) + e^{-i\sqrt{\frac{1}{3}}H}\tilde{T}_F^-(z) \\ &\equiv T_F^+(z) + T_F^-(z) \end{aligned} \tag{3.4}$$

which have charge ±1 under J:

$$J(z)T_F^\pm(z) = \frac{\pm T_F^\pm(w)}{z-w} + ... \tag{3.5}$$

Here, $\tilde{T}_F^+$ and $\tilde{T}_F^-$ are fields of conformal dimension 4/3. The fields, T_F^+ and T_F^- , satisfy the N=2 superconformal algebra, (3.1). The form of the supercurrent in (3.4) can be derived by expressing the supercurrent in a general form given by $T_F = \Sigma_q \exp[i\sqrt{1/3}Hq]\tilde{T}_F^q + \Sigma_{\hat{q}}\partial H \exp[i\sqrt{1/3}H\hat{q}]\hat{T}_F^{\hat{q}}$.

BRST invariance of the massless fermion vertex operator and dimensional analysis require that the operator product between T_F and the spin fields contain a branch cut of order 1/2,

$$T_F(z)\Sigma(w) = (z-w)^{-1/2}\tilde{\Sigma}(w) + ... \quad ; \quad T_F(z)\Sigma^\dagger(w) = (z-w)^{-1/2}\tilde{\Sigma}^\dagger(w) + ... \tag{3.6}$$

where $\tilde{\Sigma}$ and $\tilde{\Sigma}^\dagger$ are operators of dimension 11/8. Inserting the expressions for the supercurrent, $T_F = \Sigma_q \exp[i\sqrt{1/3}Hq]\tilde{T}_F^q + \Sigma_{\hat{q}}\partial H \exp[i\sqrt{1/3}H\hat{q}]\hat{T}_F^{\hat{q}}$, and the spin field, $\Sigma = e^{i\frac{\sqrt{3}}{2}H}$, into (3.6), we find that the allowed charges are $q = \pm 1$, and $\hat{T}_F^{\hat{q}} = 0$. This tells us that the supercurrent can be split into parts having definite U(1) charges as in (3.4). To show that this form leads to an N=2 superconformal algebra , we must determine the algebra generated by these currents.

The argument proceeds as discussed in [6]. For the sake of brevity, we only give a brief outline of the closure of the N=2 algebra.

It now follows that

$$J(z)T_F(w) = \frac{T_F^+ - T_F^-}{(z-w)} \equiv \frac{T_F'}{(z-w)} + ... \tag{3.7}$$

and $-2T_F'$ defined here is the upper component of a dimension one N=1 superfield whose lower component is J(z)[10]. It follows from the operator product expansion (OPE) of the superfields that $T_F(z)T_F'(z) = -(z-w)^{-2}\frac{1}{2}J(w) - (z-w)^{-1}\frac{1}{4}\partial J(w) + ...$ The OPE of $T_F'T_F'$ can be derived using the OPE'S of JJ, JT_F, JT_F', and T_FT_F'. It is given by $T_F'(z)T_F'(w) = -T_F(z)T_F(w) + ...$ Finally, using the OPE'S $T_F'T_F', T_FT_F'$, and T_FT_F, we get the operator product expansion for $T_F^+T_F^-$ given in (3.1). This closes the N=2 superconformal algebra. Therefore, any free string theory with N=1 world-sheet invariance and massless fermions,i.e., any theory with a singularity of at most $-\frac{1}{2}$ as in (3.6), requires N=2 superconformal invariance. Furthermore, we see that there is a different representation of the superconformal algebra associated with each massless fermion. N ote that this argument did not depend on any particular form of the action and is valid for any free conformal field theory with N=1 superconformal symmetry and massless fermions. The above argument also shows that in the more general case of a nontrivial space-time metric, space-time supersymmetry requires an N=2 superconformal invariance[6,7].

The vertex operators of the theory are primary fields of the N=2 superconformal algebra. The states of these theories with an extended N=2 superconformal algebra will satisfy the following conditions:

$$G_r^{\pm}|\phi>= 0 \qquad r > 0 \tag{3.8a}$$

$$L_n|\phi>= 0 \qquad n > 0 \tag{3.8b}$$

$$L_0|\phi>= h|\phi> \tag{3.8c}$$

$$J_n|\phi>= 0 \qquad n > 0 \tag{3.8d}$$

$$J_0|\phi>= q|\phi> \tag{3.8e}$$

q is the U(1) charge of the state. The conditions 3.8 b,d follow from $G_r^{\pm}|\phi>= 0$ and the N=2 algebra. Note however, that the U(1) charges of all the physical states in a theory with N=2 (gauged) world sheet supersymmetry would be zero whereas the ories with a "global" N=2 world sheet supersymmetry can have nonzero U(1) charges as in (3.10e).

If the string model with local N=1 world sheet supersymmetry has space-time supersymmetry, there are constraints on the U(1) charges of the states. Locality of the theory

requires that the operator product of any two vertex operators has no branch cuts. T he presence of the gravitino in the space-time supersymmetric case requires that the U(1) charges of all the states be integral or half-integral. In general, the complete Hilbert space, containing the left and right movers, must obey the locality conditio n. In the case of the gravitino, the left movers always contribute integer powers of $(\bar{z}-\bar{w})$ so the right movers, which contain the U(1) current J(z), must have integer powers of $(z-w)$. This is the reason that there is a quantization conditio n in the space-time supersymmetric case but not in a case without space-time supersymmetry. To demonstrate this, we will give an example of a model which has a conventional supercurrent and an N=2 superconformal symmetry, but does not have space-time sup ersymmetry.

4. N=2 supercurrents for a chiral N=1 spacetime supersymmetric model

We now show explicit realizations of the N=2 superconformal algebra for particular type II superstring models. We give a general method for finding the N=2 supercurrents provided certain criteria are met. Then, we relate the expressions given in the fermionic formalism below with the general forms given in the bosonic description in sect. 3.

We consider four-dimensional type II models in the light-cone description. The left and right movers can each be described by two bosonic and twenty fermionic fields [11-14]. An N=1 model with $SU(2)\times U(1)^5$ gauge symmetry [12-15] can be described by three generators b_0, b_1, b_2.The vectors, $\rho_{b_i} = 2(\nu_1,...\nu_n;\nu_1,...\nu_m;\nu_1,...\nu_{n'};\nu_1,...\nu_{m'})$ describing the boundary conditions of the fermi ons in the respective sectors are:
$\rho_{b_0}=((1)^{12};(1)^4;(1)^{12};(1)^4)$, $\rho_{b_1}=((0)^{12};(0)^4;(1)^4(0)^8;(0)^2(1)^2)$, and $\rho_{b_2}=((0)^{10}(1)^2;(1/2)^4;(0)^2(1)^2(0)^4(1)^4;(1/2)^4)$. The symbol h is used below to denote a real fermion that can have Ramond (R) or Neveu-Schwarz (NS) boundary conditions.

The N=1 supercurrent is given by:

$$\begin{aligned}T_F =&\frac{i}{2}h^1h^3h^7+\frac{i}{2}h^2h^4h^8+\frac{1}{2\sqrt{2}}(h^5f^1\tilde{f}^3+h^5f^3\tilde{f}^1+h^6f^4\tilde{f}^2+h^6f^2\tilde{f}^4)\\&+\frac{1}{2\sqrt{2}}(h^9\tilde{f}^1\tilde{f}^3+h^9f^3f^1+h^{10}\tilde{f}^2\tilde{f}^4+h^{10}f^4f^2).\end{aligned}\tag{4.1}$$

Using the N=2 superconformal algebra (3.1) we get T_F^+ and T_F^- in terms of T_F and T'_F:

$$T_F^+=\tfrac{1}{2}(T_F+T'_F);\qquad T_F^-=\tfrac{1}{2}(T_F-T'_F).\tag{4.2}$$

Our basic approach to finding the N=2 supercurrents is to first determine a U(1) current satisfying the N=2 superconformal algebra. This is determined from either (3.2) or (3.3). The operator product of J(z) with the N=1 supercurrent will give $T'_F(w)$ from (3.7). Then, we use (4.2) to put these N=2 supercurrents in the basis defined by T_F^+ and T_F^-.

We now use the operator products of spin fields to determine a U(1) current of the N=2 superconformal algebra. These operator products are most easily calculated using the bosonization techniques introduced in [16]. The U(1) current associated with the spin field $\Sigma(z)=e^{\frac{i}{2}\phi_1}e^{\frac{i}{2}\phi_2}e^{\frac{i}{2}\phi_3}$ is $J(z)=i\partial\phi_1(z)+i\partial\phi_2(z)+i\partial\phi_3(z)$. Since this spin field is in

the Ramond sector, the corresponding fermions will have Ramond boundary conditions. Now to relate this to the models with twisted fermions, we fermionize in the following way:

$$e^{\pm i\phi_1} = \frac{1}{\sqrt{2}}(h^1 \mp ih^2). \tag{4.3}$$

For the complex part, we bosonize the fermions in the following way:

$$e^{i\phi_2} = f^3;\ \ e^{-i\phi_2} = \tilde{f}^3;\ \ e^{i\phi_3} = f^4;\ \ e^{-i\phi_3} = \tilde{f}^4. \tag{4.4}$$

The U(1) current in fermionic form is then

$$J(z) = i : h^1(z)h^2(z) : + : f^3(z)\tilde{f}^3(z) : + : f^4(z)\tilde{f}^4(z) : -\frac{\nu_3}{z} - \frac{\nu_4}{z}. \tag{4.5}$$

The N=2 supercurrents are then found using (4.2).

$$T_F^+ = \frac{i}{4}h^1h^3h^7 + \frac{i}{4}h^2h^4h^8 + \frac{1}{4}h^2h^3h^7 - \frac{1}{4}h^1h^4h^8 + \frac{1}{2\sqrt{2}}\{f^3(\tilde{f}^1h^5 + f^1h^9) + f^4(\tilde{f}^2h^6 + f^2h^{10})\} \tag{4.6}$$

and $T_F^- = {T_F^+}^\dagger$.

These results are consistent with the general result given in sect. 3 . There, it was shown that whenever there is N=1 world-sheet supersymmetry and massless fermions are present, the supercurrent can be put in the form of $T_F = \sum_q \exp[i\sqrt{1/3}Hq]\tilde{T}_F^q$ with charges ± 1. We relate this general form to the explicit form constructed from fermionic models. We first bosonize the fermions of the U(1) current as in (4.3). The supercurrent will then be in a form given by $T_F = \Sigma_{q_i} e^{iq_1\phi_1}e^{iq_2\phi_2}e^{iq_3\phi_3}\tilde{T}_F^{(q_i)}$ where T_F has conformal dimension one and the allowed values of (q_1, q_2, q_3) are given by $(\pm 1, 0, 0)$,$(0, \pm 1, 0)$, and $(0, 0, \pm 1)$.

With the following rotation of the bosonic fields ϕ_1,ϕ_2,ϕ_3

$$\phi_1 = \frac{1}{\sqrt{3}}(H + aH_2 + bH_3); \phi_2 = \frac{1}{\sqrt{3}}(H + cH_2 + dH_3); \phi_3 = \frac{1}{\sqrt{3}}(H + eH_2 + fH_3) \tag{4.7}$$

the supercurrent takes the form:

$$T_F = e^{i\sqrt{\frac{1}{3}}H}\tilde{T}_F^+ + e^{-i\sqrt{\frac{1}{3}}H}\tilde{T}_F^-$$

$$\tilde{T}_F^+ = \frac{e^{i(aH_2+bH_3)}}{2\sqrt{2}}(-h^4h^8 + ih^3h^7) + \frac{e^{i(cH_2+dH_3)}}{2\sqrt{2}}(\tilde{f}^1h^5 + f^1h^9) + \frac{e^{i(eH_2+fH_3)}}{2\sqrt{2}}(\tilde{f}^2h^6 + f^2h^{10}) \tag{4.8}$$

and $\tilde{T}_F^- = {\tilde{T}_F^+}^\dagger$.

The a,b,c,d,e, and f describe a rotation of the bosonic fields ϕ_i which put the N=2 supercurrents and U(1) current in the desired form. The U(1) current is then given by

$J(z) = i\sqrt{3}\partial H = i\partial\phi_1 + i\partial\phi_2 + i\partial\phi_3$ and the spin field is $\Sigma = e^{i\sqrt{\frac{3}{2}}H}$. These are the expressions given in sect. 3.

5. N=2 supercurrents for a string model without space-time supersymmetry

In sect. 3, we outlined an argument that any string theory with N=1 world sheet supersymmetry and massless fermions at tree level has N=2 superconformal invariance. In this section, we show an explicit construction of the N=2 supercurrents for a superst ring model with massless fermions but without space-time supersymmetry. The model we would like to consider is given in ref.[12.13] and is generated by the sectors b_0,b_1 with $\rho_{b_0} = ((1)^{14};(1)^3;(1)^{14};(1)^3)$ and $\rho_{b_1} = ((0)^{12},(1)^2;(\frac{1}{3})^3;(1)^2,(0)^8,(1)^4;\frac{1}{3},(-\frac{2}{3})^2)$. The right moving fields can be denoted by $h^{\hat{i}}(z)$, $h^I(z)$,$f(z) = \sum_{r\in Z+\lambda} f_r z^{-r-\frac{1}{2}}$, and $f^i(z) = \sum_{r\in z+\lambda} f^i_r z^{-r-\frac{1}{2}}$, where $\hat{\imath}, i$ =1,2; and I=1,...12. The associated supercurrent has c=9 for the internal degrees of freedom and is given by

$$\begin{aligned} T_F = &-\frac{i}{2}[h^9h^1h^2 + h^{10}h^3h^4 + h^{11}h^5h^6] + \frac{1}{2\sqrt{3}}[h^{12}(\bar{h}^7\bar{h}^8 + \tilde{f}^2f^2 + f^1\tilde{f}^1) \\ &+ f(\bar{h}^7\tilde{f}^2 + \tilde{f}^1\bar{h}^8 + f^1f^2) + \tilde{f}(\bar{h}^7f^1 + f^2\bar{h}^8 + \tilde{f}^2\tilde{f}^1)] \end{aligned} \tag{5.1}$$

where $\bar{h}^7 = h^7 + ih^8$, $\bar{h}^8 = h^7 - ih^8$.

The U(1) current derived from the internal spin field in sector b_1^3 has a simple form. The boundary conditions of the fermions in this sector are given by: $\rho_{b_1^3} = ((0)^{12},(1)^2;(1)^3;$ $(1)^2,(0)^8,(1)^4;(1),(0)^2)$. In this sector, there are no twisted world-sheet fermions. The U(1) current associated with the massless fermion in sector b_1^3 is given by $J(z) = i:h^9h^{10}:$ $+i:h^{11}h^{12}: + :f\tilde{f}: -\nu z^{-1}$. The N=2 supercurrents are $T_F^- = T_F^{+\dagger}$ where

$$\begin{aligned} T_F^+ = &-\frac{1}{4}(ih^9 + h^{10})(h^1h^2 + ih^3h^4) + -\frac{1}{4}(ih^{11} + h^{12})[h^5h^6 - \frac{1}{\sqrt{3}}(\bar{h}^7\bar{h}^8 + \tilde{f}^2f^2 + f^1\tilde{f}^1)] \\ &+ \frac{1}{2\sqrt{3}}f(\bar{h}^7\tilde{f}^2 + \tilde{f}^1\bar{h}^8 + f^1f^2) \end{aligned} \tag{5.2}$$

This model does not meet all of the conditions that are required to have space-time supersymmetry. Although it has an N=2 superconformal symmetry, all the states do not have integral or half integral U(1) charges. As pointed out in sect. 3, it has N=2 su perconformal symmetry simply because it has massless fermions.

There is, in fact, a different representation of the N=2 superconformal algebra associated with every massless fermion. In other words, each internal spin field of the massless fermion vertex operator gives rise to a U(1) current belonging to a different repre sentation of the N=2 superconformal algebra. For example, the generator sector ρ_{b_1} has an internal spin field in the twisted sector. The U(1) current derived from this spin field is $J(z) = i:h^9h^{10}: +i:h^{11}h^{12}: +\frac{1}{3}:f\tilde{f}: -\nu z^{-1} + \frac{2}{3}:f^1\tilde{f}^1: -\nu_1 z^{-1} + \frac{2}{3}:f^2\tilde{f}^2: -\nu_2 z^{-1}$. This U(1) current has corresponding N=2 supercurrents which can be derived as shown above.

6. Extended N=1 supercurrent in theories with massless fermions

We now consider the new form of the N=1 fermionic supercurrent (2.1) and determine when it can be used in theories with massless fermions. In sect. 3, we showed that any supersting theory with massless fermions at tree level must form an N=2 superconforma l algebra. To be more precise, the internal supercurrent for the movers with the massless fermions must be able to be split into two parts with U(1) charges ± 1. To form the supercurrent for the movers with massless fermions, ρ_a in (2.1a) must be zero. Furthermore, the gauge symmetry for the movers with the massless fermions must be abelian, i.e. the first term in (2.1) must have, for eg., for real world sheet fermions, SU(2) structure constants, and the fermions must have mixed NS and R boundary conditions [17].

We now give a simple illustration of the N=2 supercurrents associated with the new form of the N=1 supercurrent. In this example, we give an explicit form of the supercurrent for the movers in a sector which contains a massless fermion. The fermions with space-time indices and six internal fermions will have Ramond boundary conditions. The remaining 12 internal fermions will have Neveu-Schwarz boundary conditions. In this case, the U(1) current associated with the internal spin field is given by $J(z) = \sum_{n=1}^{3} i : \psi_{(2n-1)}(z)\psi_{2n}(z) :$. The N=1 supercurrent splits into parts as $T_F = T_F^+ + T_F^-$ where $T_F^\pm$ have U(1) charges ± 1. Thus, the N=2 supercurrents associated with the new fermionic form of the N=1 supercurrent (2.1) are:

$$
\begin{aligned}
T_F^+ = \sum_{n=1}^{3} [& -\tfrac{i}{4}\psi_{(2n-1)}\psi_{(4n+3)}\psi_{(4n+4)} - \tfrac{i}{4}\psi_{(2n)}\psi_{(4n+5)}\psi_{(4n+6)} + \\
& \tfrac{1}{4}\psi_{(2n-1)}\psi_{(4n+5)}\psi_{(4n+6)} - \tfrac{1}{4}\psi_{(2n)}\psi_{(4n+3)}\psi_{(4n+4)} + \\
& \sigma^{(2n-1)}\frac{\psi_{(2n-1)}}{2z} + \sigma^{2n}\frac{\psi_{(2n)}}{2z} + i\sigma^{(2n)}\frac{\psi_{(2n-1)}}{2z} - i\sigma^{(2n-1)}\frac{\psi_{(2n)}}{2z}]
\end{aligned}
\tag{6.1}
$$

and $T_F^- = T_F^{+\dagger}$.

Furthermore, in closed superstring theories, the entire hermitian form of the supercurrent (2.1) can be used to describe the supercurrent for the movers without a massless fermion. In such a case, massless fermions can appear on the other side.

7. Conclusions and comments

The form of the supercurrent plays a critical role in determining the particle content of a string theory. This paper identifies a new extended version of the N=1 supercurrent. In order to better understand its role, we analyzed the relationship between space-time and world sheet properties. We addressed the question of which world sheet properties are needed to have a theory with massless fermions at tree level which is not nec essarily space-time supersymmetric. We found that when a free conformal field theory with a local N=1 superconformal invariance has massless fermions, a global N=2 superconformal symmetry is required and there is no quantization condition on the U(1) char ges of the states. To illustrate these general findings, we gave an example of a string model with N=2 superconformal invariance which does not have space-time supersymmetry. We then

discussed how the new fermionic construction of the N=1 supercurrent can be used in theorie s with massless fermions and we computed its corresponding N=2 supercurrents. The extended N=1 currents in (2.1) are useful in describing theories with restricted gauge symmetry. It will be interesting to incorporate this form of the superVirasoro generators in models with generalized GSO projections.

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